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S.N. Verzunov, N.M Lychenko NON-STATIONARY TIME SERIES FORECASTING BASED ON MULTIWAVELET POLYMORPHIC NETWORK

Institute of Automation and Information Technologies of NAS KR, Bishkek, Kyrgyzstan

There are many methods and models for forecasting non-stationary time series. However, the problem of the accuracy and adequacy of the forecast of non-stationary time series has not been solved yet. In this paper, a new forecast model, based on a multiwavelet network with additional customizable parameters, which is called polymorphic, is proposed. The efficiency of the proposed model is compared with the well-known time series forecast models like autoregressive integrated moving average model, multilayer perceptron and hybrid model in which both models are combined. Three well-known real data sets (the Wolf's sunspot data, the Canadian lynx data and the British pound/US dollar exchange rate data) were taken as empirical data. The comparison showed that forecast model based on the proposed multiwavelet polymorphic network has a smaller prediction error for each series. This is achieved by introducing additional customizable parameters into the wavelet network, which allow to better adapt to the non-stationary nature of time series. Moreover, for the wavelet network to perform well in the presence of linearity, were used linear connections between the wavelet neurons of input and output layers. The proposed technology can be used to predict the time series generated by dynamic processes of a different nature.

Keywords: forecasting, non-stationary time series, multiwavelet network, additional customizable parameters, ARIMA-model, artificial neural networks, hybrid model.

1. Introduction

The behavior of many processes of the real world is represented in the form of time series, i.e. sequences of values of any parameters of the process under study in discrete, equidistant moments of time. In such cases, the problem of forecasting the dynamics of the process is reduced to the problem of extrapolation of time series, based on a model built on the results of its analysis.

The desire of researchers to improve the accuracy of time series forecasting led to the existence of many methods and technologies for constructing models to approximate the original values of time series. The classic model for the analysis and prediction of non-stationary time series is the model of Boxing-Jenkins ARIMA – Autoregressive Integrated Moving Average, which combines the autoregressive model and the model of the moving average and uses the differences from the original time series [1]. The forecast of time series on the basis of this model is devoted to many works proving the effectiveness of its application in the analysis of time series of different physical nature.

However, approaches to ARIMA method suggest that time series are generated by linear processes, whereas the real-world mechanisms that generate them are often of a non-linear nature. Therefore, one of the most accurate and

widespread tools for building a forecasting model is prediction method based on the artificial neural networks (ANN). In order to recover the unknown approximating function of many variables by a set of values given by the history of the time series is used trained multilayer neural network with several inputs corresponding to these variables [2]. ANN-technologies of time series analysis and forecasting are currently devoted to many works and these technologies continue to develop. For example, in [3], neural networks (multilayer perceptron) of forward and reverse propagation are considered. The algorithms of conjugate gradients of quasi-Newton (BFGS-algorithm) were used for adjusting weights. A homogeneous ensemble technology is proposed by training neural network with each of the considered algorithms and for each algorithm the weight is calculated (inversely proportional to the forecast error) and final forecast is calculated as the weighted arithmetic mean of all forecasts. In [4], the ANN is applied to forecast multi-scale Internet traffic and comparison of the effectiveness of the Holt-Winter, ARIMA and ANN models is showed. The paper [5] discusses two ways to solve the problem of forecasting demand in seasonal time series using ANN. In first method multilayer model of the perceptron was used, where input is fed from the previous time series values. Several learning rules used to correct the weights of a multilayer perceptron (inverse error propagation, adaptive and Levenberg-Marquardt). In the second method, a causal method-based ANN was used, where input is taken from decomposed time series components (trend, seasonal and random components).

Much attention of researchers attracted to ANN forecasting technology that combines different models, so-called hybrid or ensemble model. Hybrid models can be homogeneous, using different neural networks (all multilayer perceptrons, for example) or heterogeneous, for example, combining both linear and nonlinear models. The main idea behind this multi-model approach is to use each component's unique model ability to improve the capture of different patterns in the data. For example, [6, 7, 8] show improvement of forecasting characteristics of each individual model by combining ARIMA and ANN based models. The proposed in [9] hybrid method is inspired from a similar concept, pioneered by Zhang [6] and suggests to improve the forecasting accuracy of hybrid models by segregating a time series dataset into linear (detailed) and nonlinear (approximate) components through discrete wavelet transform. After dataset decomposition ARIMA and ANN models are used to separately recognize and forecast the detailed and approximate components, respectively.

We note that the use of wavelet transforms in solving forecast problems of various, especially non-stationary time-series, where the spectral content changes with time, seems to be attractive to many researchers. This can be explained by the fact that wavelets elements are well localized both in the time and in the frequency domains [10, 11]. The wavelet transform decomposes the main time series into subcomponents, which allows us to consider and capture useful information at different resolution levels. Various wavelet-based ARIMA models and combined wavelet-artificial neural network models were proposed [12, 13, 14]. These models provide a higher prediction accuracy than conventional ARIMA and ANN models.

In recent years, specialized neural networks or wavelet networks [15] have been proposed. The wavelet networks are modification of networks, based on radial basis functions were wavelets are used as basic functions. Various structures of wavelet networks are proposed; however, their general idea is to adjust the parameters of compression and shift of wavelet neurons for the best data learning. So, in [16] a multiwavelet neural network is proposed, in which a multidimensional wavelet is used as an activation function of wavelet neurons in a hidden layer, that allows to approximate multidimensional functions.

The paper [17] proposes a new structure of wavelet networks for time series processing, which differs from the traditional one by introducing additional configurable parameters into the basic wavelets. Such a network, called polymorphic that has better approximating properties due to better adaptability to the character of time series non-stationarity. In [18] to account for inertia in time series, inverse relations are introduced into the structure of polymorphic wavelet network, considering the time series levels in the previous moments of time.

In this paper, in order to improve the accuracy of prediction of nonstationary time series, a new model of the wavelet network is proposed, which combines the advantages of the wavelet networks considered in [17, 18, 19]. To illustrate the efficiency of the proposed model, computational experiments were carried out with three well-known data: the Wolf's sunspot data, the Canadian lynx data and the British pound/US dollar exchange rate data. A comparison of the obtained results with the forecast results on ARIMA, ANN and hybrid ANN models [6] are presented to show that the proposed new model has advantages over the known models in terms of reducing forecast errors.

Next, the paper is organized as follows: the following section presents a new solution for the prediction of nonstationary time series, which is model of a multivalued polymorphic wavelet network. In section 3, on the basis of the proposed model the forecast for three known time series is carried out and the results are compared with the forecast results obtained on other models (ARIMA, ANN, hybrid ANN). Finally, section 4 contains the conclusion.

2. Forecast model based on a multiwavelet polymorphic network

As mentioned above, wavelet network is a three-layer neural network in which the first layer is input layer, the second layer is hidden layer and the third layer is output layer. Various structures of wavelet networks are proposed; however, their general idea is to adjust the parameters of compression and shift of wavelet neurons for the best data learning. Thus, in [16], the structure of a multiwavelet neural network is proposed, where a multidimensional wavelet is used as an activation function of wavelet neurons in a hidden layer, which allows to approximate multidimensional functions. Also, in the works [17, 18] a polymorphic wavelet network is proposed, characterized by the presence of an additional configurable parameter of the mother wavelet and allowing more accurate approximation of nonstationary time series. In this paper, we proposed a new structure of the wavelet network that combines the advantages of the wavelet networks considered in [17, 18, 19].

The output of the traditional multiwavelet network [12] is determined by the equation:

$$\hat{y}(\mathbf{x}) = g_{\lambda}(\mathbf{x}; \mathbf{w}) = w_{\lambda+1}^{[2]} + \sum_{j=1}^{\lambda} w_j^{[2]} \cdot \Psi_j(\mathbf{x}) + \sum_{i=1}^m w_i^{[0]} \cdot x_i, \qquad (1)$$

where $\Psi j(\mathbf{x})$ – a multidimensional wavelet which is set as the product of m scalar wavelets, \mathbf{x} – the input data vector, m – number of inputs, λ – the number of hidden wavelet neuron and \mathbf{w} – parameters of a network: $w^{[2]}_{\lambda+1}$ – weight bias, $w^{[0]}_i$ – weight linear relationships and $w^{[2]}_j$ – weight of non-linear relationships. The multidimensional wavelet in formula (1) is calculated with

$$\Psi_{j}(\mathbf{x}) = \prod_{i=1}^{m} \psi(z_{ij}), \qquad (2)$$

where ψ is the mother wavelet and

$$z_{ij} = \frac{x_i - w_{(\xi)ij}^{[1]}}{w_{(\zeta)ij}^{[1]}}.$$
(3)

In the expression (3) $i=1, ..., m, j=1, ..., \lambda+1, w^{[1]}_{(\zeta)ij}$ – parameters of shift and $w^{[1]}_{(\zeta)ij}$ – parameters of the scale of the wavelets.

Mother wavelets commonly used as first derivative of the Gaussian of the so-called "WAVE-wavelet":

$$\psi(z_{ij}) = z_{ij}e^{-0.5z_{ij}^2},$$
(4)

the second derivative of Gaussian so-called "Mexican hat":

$$\psi(z_{ij}) = (1 - z_{ij}^2) e^{-0.5 z_{ij}^2}, \qquad (5)$$

or wavelet Morlet:

$$\psi(z_{ij}) = \cos(5z_{ij})e^{-0.5z_{ij}^2}.$$
(6)

The choice of the mother wavelet depends on the problem to be solved and is not limited to the above functions, in particular, it is possible to use orthogonal wavelets and wavelet frames.

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In multiwavelet polymorphic network (Figure 1) mother wavelets are used with an additional adjustable parameter that changes the shape of the wavelet in a way different from compression and shift [17]. For example, we can use the polymorphic mother wavelet Superposed LOGistic functions ("logistics functions superposition"), known as SLOG:

$$\psi(z_{ij}, w_{(\rho)ij}^{[1]}) = \frac{1}{1 + e^{-z_{ij} + w_{(\rho)ij}^{[1]}}} - \frac{1}{1 + e^{-z_{ij} + 3w_{(\rho)ij}^{[1]}}} - \frac{1}{1 + e^{-z_{ij} - 3w_{(\rho)ij}^{[1]}}} + \frac{1}{1 + e^{-z_{ij} - w_{(\rho)ij}^{[1]}}}$$

where $w^{[1]}_{(\rho)ij}$ is an additional customizable parameter of the wavelet shape that determines the rate of attenuation. Then (2) will look like:

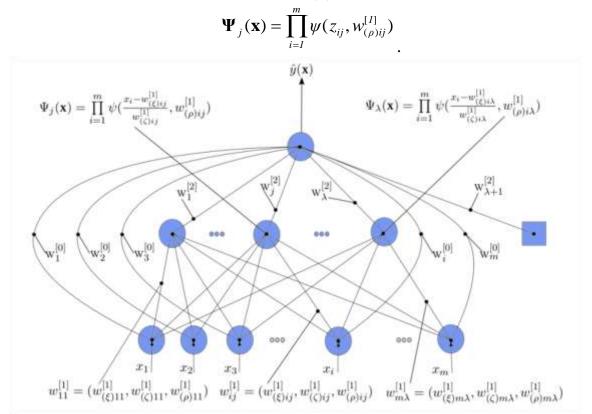


Figure 1 - A multiwavelet polymorphic network

In this study we implement a multiwavelet polymorphic network with a linear connection between the wavelet neuron of input and output layers whith help weight linear relationships $w^{[0]}_i$ (Figure 1). Thus, the wavelet network combines the advantages of AR-models and ANN, due to the presence of linear $w^{[0]}_i$ and non-linear $w^{[2]}_i$ relationships.

The full vector of network parameters includes $\mathbf{w} = (w^{[0]}_i, w^{[2]}_j, w^{[2]}_{\lambda+I}, w^{[1]}_{(\zeta)ij}, w^{[1]}_{(\zeta)ij}, w^{[1]}_{(\rho)ij})$. These settings are configured during the process of network training by minimizing the root mean square error

where *n* is the number used for the network training examples, y_p is the desired value of network output and \hat{y}_p is the actual value of the network output. The root mean square error is minimized by one of the iterative methods of multidimensional optimization [20] using partial derivatives of the root square error on the network parameters:

$$\frac{\partial E}{\partial w} = \frac{1}{n} \sum_{p=1}^{n} -e_p \frac{\partial \hat{y}_p}{\partial w}, \qquad (8)$$

where $e_p = y_p - \hat{y}_p$. Using the expression (8), we can find partial derivatives of the root mean square error for all network parameters, followed by formulas to re-calculate the network parameters at each iteration [19]:

$$\mathbf{w}_{v+I} = \mathbf{w}_{v} - \eta \frac{\partial E}{\partial w_{v}} + \kappa (\mathbf{w}_{v} - \mathbf{w}_{v-I}), \qquad (9)$$

where v is the number of iterations, η is the learning speed parameter and κ is the moment parameter. The specific values of the parameters η and κ are dependent on the iterative method of multidimensional optimization.

The proposed forecast method, as well as most methods related to time series processing, based on the construction of delay vectors:

$$\mathbf{x}_n = (x_n, x_{n+1}, \dots, x_{n+p-1})^T, n = 1, 2, \dots, N - p$$

and the construction of the target vector

$$\mathbf{y} = (x_{p+1}, x_{p+2}, ..., x_N),$$

where N is the number of time series counts, p is the number of delays.

Delay vectors and target vector are used to train a multiwavelet polymorphic network. In particular, delay vectors \mathbf{x}_n are fed to the input of a multiwavelet polymorphic network and then response vector $\hat{\mathbf{y}}$ is formed

$$\hat{y}_n(\mathbf{x}_n) = g_\lambda(\mathbf{x}_n, \mathbf{w}_v) \, .$$

Then, based on the values of the vectors \mathbf{y} and $\mathbf{\hat{y}}$, the root mean square error of the network is determined by the formula (7). This error is minimized using the one of the iterative methods of multidimensional optimization (in the examples given – the Broyden-Fletcher-Goldfarb-Shanno method). The network parameters are adjusted using the training rule (9).

To predict the time series with the help of a trained multiwavelet polymorphic network, an iterative method is used, consisting the sequential receipt of the forecast by one step (time interval) with the addition of its result to the original data. First, the original delay vector $\hat{\mathbf{x}}_{I} = (x_{N-p+1}, x_{N-p+2}, ..., x_{N})^{T}$ is built, on the basis of which a multiwavelet polymorphic network makes a prediction for one step: $\hat{x}_{N+I} = g_{\lambda}(\hat{\mathbf{x}}_{I}; \hat{\mathbf{w}})$. Then the resulting value is added to the original delay vector and a new vector $\hat{\mathbf{x}}_{2} = (x_{N-p+2}, x_{N-p+3}, ..., \hat{x}_{N+I})^{T}$ is built. After that the forecast is made for another step $\hat{x}_{N+2} = g_{\lambda}(\hat{\mathbf{x}}_{2}; \hat{\mathbf{w}})$ and so on:

$$\hat{x}_{(k+1)} = g_{\lambda}(\hat{\mathbf{x}}_k; \hat{\mathbf{w}}),$$

where $\hat{\mathbf{w}}$ are the parameters of the trained multiwavelet polymorphic network, k=1,2,...,K, and K is the number of predicted time series counts.

3. The results of computational experiments and discussion

In order to test the efficiency and effectiveness of the proposed forecast model, three time series were taken as experimental data (Table 1) from well-known in statistical studies the Time Series Data Library (TSDL) repository [21]: the Wolf's sunspot data, the Canadian lynx data and the British pound/United States dollar exchange rate data. Only the one-step-ahead forecasting is considered.

Series	Sample size	Training set	Test set (size)
		(size)	
Sunspot	288	1700-1920 (221)	1921–1987 (67)
Lynx	114	1821–1920 (100)	1921–1934 (14)
Exchange rate	731	1980-1992 (679)	1993 (52)

 Table 1 - Sample compositions in three data sets

MAD (Mean Absolute Deviation) and MSE (Mean Squared Error) were used to compare the forecasting effectiveness of different models:

$$MAD = \frac{1}{K} \sum_{i=1}^{K} |e_i - \overline{e}|, \qquad MSE = \frac{1}{K} \sum_{i=1}^{K} (e_i)^2,$$

where $e_p = y_i - \hat{y}_i$, *K* is the prediction interval, *y* is the actual value of the series level, \hat{y} is the forecast value of the series level. Table 2 and Figure 2 show the results of forecasting of Wolf's sunspot data. Two forecasting periods were used, 35 and 67 years. ARIMA model $9 \times 0 \times 0$ was used for the forecast. As shown in [6], this model is the most minimalistic among all ARIMA models generating close - to-magnitude prediction errors and often used in many other studies, for example in [8, 22]. Also, an ANN neural network model containing $4 \times 4 \times 1$ neurons was used for the forecast, which was substantiated in [6, 22].

 Table 2 - Forecasting comparison for sunspot data

35 points ahead	67 points ahead
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	MSE	MAD	MSE	MAD
ARIMA	216.965	11.319	306.08217	13.033739
ANN	205.302	10.243	351.19366	13.544365
Zhang'sHybrid	186.827	10.831	280.15956	12.780186
Model				
MWPN	151.825	9.1351	272.49066	12.420121

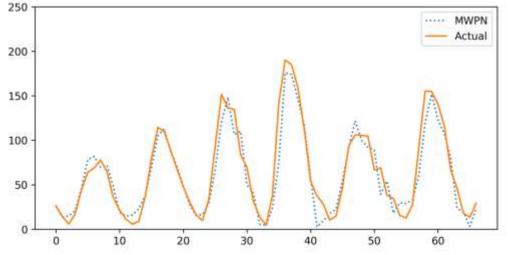


Figure 2 - Forecasting Wolf's sunspot with a multiwavelet polymorphic network

Comparison of the results shows that when using a multiwavelet polymorphic network (MWPN) containing $9 \times 4 \times 1$ wavelet neurons for forecasting, MSE predictions can be improved by 18.735% and 2.737%, respectively, compared to the Zhang's Hybrid Model [6].

Similarly, the Canadian lynx data was processed. The forecast using ARIMA is performed on the model of the order of 12 X 0 X 0. This minimalistic model was also used in [6,8]. A neural network containing 7 X 5 X 1 neurons [6] gives a slightly better prediction, compared to the ARIMA model. Figure 3 shows the results of the prediction using a multiwavelet polymorphic network containing 2 X 7 X 1 neurons. A comparison of the results (Table 3, Figure 3) shows that when using a multiwavelet polymorphic network, MSE predictions can be improved by 54.889% compared to the Chang hybrid model [6]. Here, the natural logarithm of the original data was used in constructing the model.

	MSE	MAD
ARIMA	0.020486	0.112255
ANN	0.020466	0.112109
Zhang's Hybrid model	0.017233	0.103972
MWPN	0.007774	0.063614

Table 3 - Lynx forecasting results.

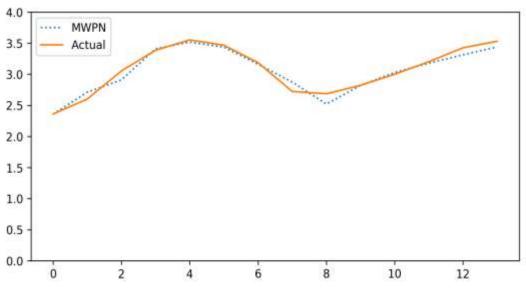


Figure 3- Forecasting of the Canadian lynx by a multiwavelet polymorphic network

The forecast for the British pound / United States dollar exchange rate data was performed for the prediction interval of 1, 6 and 12 months using the ARIMA $0 \times 1 \times 0$ model, the neural network ANN model containing $7 \times 6 \times 1$ neurons, the Zhang's Hybrid Model [6] and a multiwavelet polymorphic network containing $1 \times 7 \times 1$ neurons (Fig. 4). Comparison of the results (Table 4, Figure 4) shows that using a multiwavelet polymorphic network, MSE predictions can be improved compared to the hybrid model by 41.768%, 16.496% and 22.917%, respectively.

	1 month		6 months		12 months	
	MSE	MAD	MSE	MAD	MSE	MAD
ARIMA	3.68493	0.005016	5.65747	0.0060447	4.52977	0.005359
ANN	2.76375	0.004218	5.71096	0.0059458	4.52657	0.005251
Zhang's	2.67259	0.004146	5.65507	0.0058823	4.35907	0.005121
Hybrid						
model						
MWPN	1.55629	0.002873	4.72234	0.0052145	3.31015	0.004231

Table 4 - Exchange rate forecasting results (All MSE values should be multiplied by 10⁻⁵)

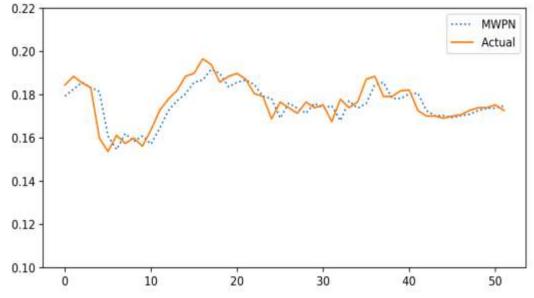


Figure 4 - Forecasting the British pound / United States dollar exchange rate with a multiwavelet polymorphic network

4. Conclusion

In this paper, the structure of a multiwavelet polymorphic network and the technology of forecasting non-stationary time series based on it are presented. The examples of short-term forecasts of known time series showed that the accuracy obtained with forecast model based on multiwavelet polymorphic network exceeds the accuracy obtained with models ARIMA, ANN and hybrid ANN (combines the ARIMA and ANN). This is achieved due to the introduction in the multiwavelet network of some special additional customizable parameters and, as a result, better adaptability is reached. Moreover, for the wavelet network to perform well in the presence of linearity, we use linear connections between the wavelet neurons of input and output layers. The proposed technology can be used to predict the time series forecasting generated by dynamic processes of a different nature.

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С.Н. Верзунов, Н.М. Лыченко ПРОГНОЗИРОВАНИЕ НЕСТАЦИОНАРНЫХ ВРЕМЕННЫХ РЯДОВ НА ОСНОВЕ МУЛЬТИВЕЙВЛЕТНОЙ ПОЛИМОРФНОЙ СЕТИ

Институт автоматики и информационных технологий НАН КР, Бишкек, Кыргызстан

Для прогнозирования нестационарных временных рядов существует много методов и моделей, однако, проблема точности и адекватности прогноза таких рядов по-прежнему является актуальной. В настоящей статье предложена новая модель прогноза, основанная на мультивейвлетной сети с дополнительными настраиваемыми параметрами, названной полиморфной. Эффективность предложенной модели сравнена с хорошо известными моделями прогноза временных рядов: моделью авторегрессионного интегрированного скользящего среднего, многослойным персептроном и гибридной моделью, комбинирующей обе указанные модели. В качестве экспериментальных данных были использованы три реальных, хорошо известных в статистике временных ряда: данные о солнечных пятнах Вольфа, данные о популяции канадской рыси и данные об обменном курсе британского фунта к доллару США. Сравнение показало, что предложенная модель прогноза на основе мультивейвлетной полиморфной сети обладает меньшей ошибкой прогноза для всех рассмотренных рядов. Это достигнуто благодаря введению дополнительных настраиваемых параметров в вейвлетсеть, которые позволяют лучше адаптироваться к нестационарной природе временных рядов. Кроме того, наличие в структуре предложенной вейвлет-сети прямых связей между вейвлет-нейронами входного и выходного слоев улучшает ее прогностические свойства для временных рядов, имеющих линейную составляющую. Предложенная технология может быть использована для прогноза временных рядов, генерируемых динамическими процессами различной физической природы.

Ключевые слова: прогнозирование, нестационарные временные ряды, мультивейвлетная сеть, дополнительные настраиваемые параметры, ARIMA-модель, искусственная нейронная сеть, гибридная модель.